Abstract

This work is concerned with two different problems in harmonic analysis, one on the Heisenberg group and other on $\mathbb{R}^n$, as described in the following two paragraphs respectively.

Let $\mathbb{H}^n$ be the $(2n+1)$-dimensional Heisenberg group, and let K be a compact subgroup of $U(n)$, such that $(K, \mathbb{H}^n)$ is a Gelfand pair. Also assume that the K-action on $\mathbb{C}^n$ is polar. We prove a Hecke-Bochner identity associated to the Gelfand pair $(K, \mathbb{H}^n)$. For the special case $K = U(n)$, this was proved by Geller, giving a formula for the Weyl transform of a function f of the type $f = Pg$, where g is a radial function, and P a bigraded solid $U(n)$-harmonic polynomial. Using our general Hecke-Bochner identity we also characterize (under some conditions) joint eigenfunctions of all differential operators on $\mathbb{H}^n$ that are invariant under the action of K and the left action of $\mathbb{H}^n$.

We consider convolution equations of the type $f * T = g$, where $f, g \in L^p(\mathbb{R}^n)$ and T is a compactly supported distribution. Under natural assumptions on the zero set of the Fourier transform of T, we show that f is compactly supported, provided g is.